Updated Walras model

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Abstract

Walras theory is well known and widely used in models of market economy [3]. Various iterative methods are developed to search for the equilibrium conditions. In this paper a search for the Nash equilibrium [2] is defined as a stochastic global optimization problem. The improved version of web-based optimization software is presented. The specific property is simulation of stochastic customer arrivals and stochastic service times and application of Bayesian Approach to optimization [1].

1 Problem formulation

Denote service price $y = (y_i, i = 1, \ldots, m)$, server resource $p = (p_i, i = 1, \ldots, m)$, amount of resource $x = (x_{ij}, i, j = 1, \ldots, m)$. Here x_{ii} are local resources, x_{ij} are server j resources used by i. Profit of server i

$$u_i(y, p, x) = a_i y_i + p_i \sum_{i \neq j} x_{ji} - \sum_{i \neq j} p_j x_{ij} \tag{1}$$

Customer expenses

$$h_i = y_i + \gamma_i, \gamma_i = \frac{n_i}{w_i}, w_i = c_{0i}(1 - e^{-c_{ii}x_{ii} - c_{ij}x_{ij}}) \tag{2}$$

where w_i is capacity of server i, b_i is resource of server i, c_{ij} defines efficiency of resources, here local resources x_{ii} are defined by balance condition:

$$x_{ii} + \sum_j x_{ij} = b_i, i = 1, \ldots, m \tag{3}$$

In two server case denote the contract resources as x_{12}^- and x_{21}^-. Then "no-contract" resources:

$$x_{12}^{=}(y, p) = \arg\max_{x_{12}} u_1(y, p, x_{12}, x_{21}^-) \tag{4}$$

$$x_{21}^{=}(y, p) = \arg\max_{x_{21}} u_1(y, p, x_{21}, x_{12}^-) \tag{5}$$

Denote

$$\begin{aligned} f_x(y, p, x_{12}^-, x_{21}^-) = u_1(y, p, x_{12}^{=}, x_{21}^-) - u_1(y, p, x_{12}^-, x_{21}^-) + \\ u_2(y, p, x_{21}^{=}, x_{12}^-) - u_2(y, p, x_{21}^-, x_{12}^-) \end{aligned} \tag{6}$$

Contract $(x_{12}^-(y, p), x_{21}^-(y, p))$ is stable if

$$\min_{x_{12}^-, x_{21}^-} f_x(y, p, x_{12}^-, x_{21}^-) = 0 \tag{7}$$

Optimal resources: $x^*_{12} = x_{12}(y, p)$ and $x^*_{21} = x_{21}(y, p)$.
Denote the contract prices $(y^-_1, p^-_1, y^-_2, p^-_2)$. Then the "no-contract" prices:

$$(y^=_1, p^=_1) = \arg\max_{(y_1,p_1)} u_1(y_1, y^-_2, p_1, p^-_2, x_{12}(y^-_2, p^-_2), x_{21}(y_1, p_1)) \tag{8}$$

$$(y^=_2, p^=_2) = \arg\max_{(y_2,p_2)} u_2(y_2, y^-_1, p_2, p^-_1, x_{12}(y_2, p_2), x_{21}(y^-_1, p^-_1)) \tag{9}$$

$$\min_{y^-_1, p^-_1, y^-_2, p^-_2} f(y^-_1, p^-_1, y^-_2, p^-_2) = 0 \tag{10}$$

$$\begin{aligned} f(y^-_1, p^-_1, y^-_2, p^-_2) = {} & u_1(y^=_1, y^-_2, p^=_1, p^-_2, x_{12}(y^-_2, p^-_2), x_{21}(y^=_1, p^=_1)) - \\ & u_1(y^-_1, y^-_2, p^-_1, p^-_2, x_{12}(y^-_2, p^-_2), x_{21}(y^-_1, p^-_1)) + \\ & u_2(y^=_2, y^-_1, p^=_2, p^-_1, x_{12}(y^=_2, p^=_2), x_{21}(y^-_1, p^-_1)) - \\ & u_2(y^-_2, y^-_1, p^-_2, p^-_1, x_{12}(y^-_2, p^-_2), x_{21}(y^-_1, p^-_1)) \end{aligned} \tag{11}$$

2 Software

The software is designed as a tool for web-based graduate studies and scientific cooperation: `http://soften.ktu.lt/~mockus`.
The difference from the software developed by P. Treigys [1] is improved prediction of competitors response to changes in resource and service prices. Figure 1 illustrates that.

3 Conclusions

The results show possibility to simulate competitive behavior by simple optimization model. The large amount of computation limits the number. of objects. Thus, the multi-processor implementation is planned.

References

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Figure 1: First server profit u_1 and resource sale x_{21} as function of service and resource prices y_1, p_1