"Portfolio" Problem, Optimal Investment of Resources

1 Introduction

The previous examples illustrated competition and inspection processes in economical, social and ecological problems. Here the optimal investment of available resources is considered. Investment problems depend on the nature of resources to be invested. An important part of any investment problem is a proper definition of utility functions that determine the profit-to-risk relation. Here we consider an illustrative example how to invest some fixed capital in Certificates of Deposit (CD) and Stocks.

The portfolio problem is to maximize the average utility of a wealth. That is obtained by optimal distribution of available capital between different objects with uncertain parameters [3]. Denote by x_i the part of the capital invested into an object i. The returned wealth is $y_i = c_i x_i$. Here $c_i = 1 + \alpha_i$ and $\alpha_i > 0$ is an interest rate. Denote by $p_i = 1 - q_i$ the reliability of investment. Here q_i is the insolvency probability. $u(y)$ is the utility the wealth y. Denote by $U(x)$ the expected utility function. $U(x)$ depends on the capital distribution $x = (x_i, ..., x_n),\ \sum_i = 1,\ x_i \geq 0$. If y is continuous, the expected utility function

$$U(x) = \mathbf{E}u(y) = \int_0^\infty u(y)p(y)dy. \quad (1)$$

Here $p(y)$ is probability density of wealth y. If the wealth is discrete $y = y^k,\ k = 1, ..., M$, the expected utility function

$$U(x) = \sum_{k=1}^{M} u(y^k)p(y^k). \quad (2)$$

Here M is the number of discrete values of wealth y^k. $p_x(y^k)$ is the probability that the wealth y^k will be returned, if the capital distribution is x. We search for such capital distribution x which provides the greatest expected utility of the returned wealth:

$$\max_x U(x), \quad (3)$$

$$\sum_{i=1}^{n} x_i = 1, \quad (4)$$

$$x_i \geq 0.$$

2 Expected Utility

2.1 Investment in CD

One may define probabilities $p(y^j)$ of discrete values of wealth $y^j,\ j = 1, 2, ...$ by exact expressions. For example,

$$\begin{aligned} p(y^0) &= \prod_i q_i, \\ p(y^1) &= p_1 \prod_{i \neq 1} q_i, \\ p(y^2) &= p_2 \prod_{i \neq 2} q_i, \\ \cdots\cdots &\quad \cdots\cdots\cdots\cdots \\ p(y^n) &= p_n \prod_{i \neq n} q_i, \\ p(y^{n+1}) &= p_1 p_2 \prod_{i \neq 1, i \neq 2} q_i, \\ p(y^{n+2}) &= p_1 p_3 \prod_{i \neq 1, i \neq 3} q_i \\ \cdots\cdots &\quad \cdots\cdots\cdots\cdots\cdots\cdots \end{aligned} \tag{5}$$

Here
$y^0 = 0,\ y^1 = a_1 x_1,\ y^2 = a_2 x_2,\ y^n = a_n x_n,$
$y^{n+1} = a_1 x_1 + a_2 x_2,\ y^{n+2} = a_1 x_1 + a_3 x_3$. From expression (5)

$$U(x) = \sum_{k=1}^{M} u(y^k) p(y^k). \tag{6}$$

Here M is the number of different values of wealth y.

One determines $U(x)$ approximately by the Monte Carlo approach:

$$U_K(x) = 1/K \sum_{k=1}^{K} u(y^k). \tag{7}$$

Here

$$y^k = \sum_{i=1}^{n} y_i^k, \tag{8}$$

where

$$y_i^k = \begin{cases} c_i x_i, & \text{if } \eta_i^k \in [0, p_i] \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Here K is the number of Monte Carlo samples. η_i^k is a random number uniformly distributed on the unit interval. In this case

$$U(x) = \lim_{K \to \infty} U_K(x).$$

2.2 Investment in CD and Stocks

Investing in CD, the interests α_i are defined by contracts. Only the reliabilities $p_i,\ i = 1, ..., n$ of banks are uncertain. Investing in stocks, in addition to reliabilities $p_i,\ i = n + j,\ j = 1, ..., m$ of companies, their future stock rates are uncertain, too. The predicted stock rates are defined by a coefficient a_i that shows the relation between the present and the predicted stock rates. The prediction "horizon" is supposed to be the same as the maturity time of CD.

To simplify the model suppose that one predicts L different values of relative stock rates $a_i^l,\ l = 1, ..., L$ with corresponding estimated probabilities $p_i^l,\ \sum_{l=1}^{L} p_i^l = 1,\ p_i^l \geq 0$.
In this case, one may define probabilities $p(y^i)$ of discrete values of wealth $y^i,\ i = 1, ..., n + m$ by exact expressions. The expressions for CD remains the same. Therefore, we shall consider only stocks assuming that $n = 0$ and $L = 2$. Then

$$\begin{aligned}
p(y^0) &= \prod_i q_i, \\
p(y^1) &= p_1 p_1^1 \prod_{i \neq 1} q_i, \\
p(y^2) &= p_1 p_1^2 \prod_{i \neq 1} q_i, \\
p(y^3) &= p_2 p_2^1 \prod_{i \neq 2} q_i, \\
p(y^4) &= p_2 p_2^2 \prod_{i \neq 2} q_i, \\
\cdots\cdots &\quad \cdots\cdots\cdots\cdots \\
p(y^{2n-1}) &= p_n p_n^1 \prod_{i \neq n} q_i, \\
p(y^{2n}) &= p_n p_n^2 \prod_{i \neq n} q_i, \\
p(y^{2n+1}) &= p_1 p_1^1 p_2 p_2^1 \prod_{i \neq 1, i \neq 2} q_i,
\end{aligned}$$

$$p(y^{2n+2}) \;=\; p_1 p_1^2 p_2^p 2^2 \prod_{i \neq 1, i \neq 2} q_i, \qquad (10)$$

....

Here
$y^0 = 0,\ y^1 = a_1^1 x_1,\ y^2 = a_1^2 x_1,\ y^3 = a_2^1 x_2, y^4 = a_2^2 x_2,$
$y^{2n-1} = a_n^1 x_n,\ y^{2n} = a_n^2 x_n,\ y^{2n+1} = a_1^1 x_1 + a_2^1 x_2,\ y^{2n+2} = a_1^2 x_1 + a_2^2 x_2$. The reliabilities p_i, the stock rate predictions a_i^l and their estimated probabilities p_i^l are defined by experts, possibly, with the help of time series models such as ARMA For example, maximal values of multi-step prediction are considered as "optimistic" estimates. The minimal values- as "pessimistic" ones. The average values of multi-step prediction are regarded as "realistic" estimates.

Here is a simplest illustration were $n = m = 1$ and $L = 2$. In this case from (5) (10) the probabilities $p(y^k)$ of wealth returns $y^k,\ k = 0, ..., 5$ are

$$\begin{aligned}
p(y^0) &= q_1 q_2, \\
p(y^1) &= p_1 q_2, \\
p(y^2) &= p_2 p_2^1 q_1, \\
p(y^3) &= p_2 p_2^2 q_1, \\
p(y^4) &= p_2 p_2^1 p_1, \\
p(y^5) &= p_2 p_2^2 p_1.
\end{aligned}$$

Here
$y^0 = 0,\ y_1 = a_1 x_1\ y^2 = a_2^1 x_2,\ y^3 = a_2^2 x_2,$
$y^4 = a_1 x_1 + a_2^1 x_2,\ y^5 = a_1 x_1 + a_2^2 x_2.$

3 Optimal Portfolio, Special Cases

The optimal portfolio depends on the utility function $u(y)$. Consider, for example, the optimal portfolio for three different utility functions.

The first utility function is linear

$$u(y) = cy. \qquad (11)$$

This function is for "rich" persons. Rich persons want to maximize the average wealth. They are not emotional about accidental losses or gains. In the linear case (11), the optimal portfolio is to invest all the capital in an object with the highest product $p_i c_i$.

The second utility function is for "prudent" persons which averse risk

$$u(y) = \begin{cases} 0 & \text{if } 0 \leq y < a \\ 1 & \text{if } a \leq y \leq c \end{cases} . \tag{12}$$

Here a is a risk threshold. $c = \max_i c_i x_i$ denotes the maximal return of invested capital (see expression (4)). If $a = 1/m \ min_i \ c_i x_i$ then, in the risk-averse case the optimal decision is $x_i^* = 1/m, \ i = 1, ..., m$. Here one divides the capital equally between all the objects[1].

The third utility function is for "risky" persons. Risky persons are ready to risk for the great win c.

$$u(y) = \begin{cases} 0 & \text{if } 0 \leq y < c \\ 1 & \text{if } y = c \end{cases} . \tag{13}$$

Here one invests all the capital in the object with highest wealth return. Therefore, $x_i = 1$, if $c_i = \max_j c_j = c$.

These examples are abstract. An average person behaves "risky," if only a small part of his resources is involved. The same person behaves prudently, if all his wealth is at stake. There is a point r between areas of risky and prudent behavior. At this point an average person behaves like the "rich" one. Here is an example

$$\begin{aligned} u(y) < y, \ if \ 0 \leq y < r, \\ u(y) = y, \ if \ y = r, \\ u(y) > y, \ if \ r < y \leq c. \end{aligned} \tag{14}$$

Here r is a boundary point between risky and prudent areas.

4 Optimal Insurance

In this section the optimal insurance is regarded as a special case of optimal investment. Then the expected utility at the end of the next year

$$U(x, a) = \sum_{k=1}^{M} u(y^k) p(y^k), \tag{15}$$

[1]The optimal decision $x = 1/m$ is not unique, any decision satisfying the inequality $c_i x_i \geq a, \ i = 1, ..., m$ minimizes the expected utility function $U(y)$.

where $p(y^k)$ is a probability to get a wealth y^k,
$u(y^k)$ is an utility function of this wealth y^k.
Here $a = (a_i, \ i = 1, ..., m)$ is a vector which components a_i define rates of insurance charges for different objects $i = 1, ..., m$. Suppose that
$y^k = \sum_{i=1}^m c_i(x_i)$
and

$$c_i(x_i) = \begin{cases} -a_i x_i & \text{if } \delta_i = 1 \\ (1 - a_i)x_i & \text{if } \delta_i = 0 \end{cases}. \tag{16}$$

Here [2] the product $a_i x_i$ denotes insurance charge of the object i,
$x_i \leq z_i$ is insurance policy of the object i,
z_i is market value of the object i,
$\delta_i = 1$, if the object i survives,
$\delta_i = 0$, otherwise,
$p_i = P\{\delta_i = 1\}$ is a survival probability of the object i.
For example:
$p(y^1) = p_1 \prod_{i \neq 1}(1 - p_i)$,
$y^1 = c_1(x_1) + \sum_{i=2}^m c_i(x_i)$,
where
$c_1(x_1) = z_1 - a_1 x_1$,
$c_i(x_i) = (1 - a_i)x_i, \ i = 2, ..., m$.

5 Nash Equilibrium

5.1 Single Company Insuring Single Object

In this section the optimal insurance is regarded as a problem of insurance company. We start from the simplest case of single company insuring a single object.

The expected utility of this object

$$U(x, a) = \sum_y u(y)p(y), \tag{17}$$

[2]In the early models it was supposed that $c_(x_i) = z_i - a_i x_i$, here we omit the first component because market value z_i of the object i is fixed independently of optimization results.

where $p(y)$ is a probability of wealth y,
$u(y)$ is an utility function of the wealth y.
Here a is the rate of insurance charge.

Suppose that
$y = c(x)$
and

$$c(x) = \begin{cases} -ax & \text{if } \delta = 1 \\ (1-a)x & \text{if } \delta = 0 \end{cases}. \qquad (18)$$

Here z is the market value of the object,
the product ax denotes insurance charge of the object.
$x \leq z$ is insurance policy of the object.
$\delta = 1$, if the object survives,
$\delta = 0$, if not.
$p = P\{\delta = 1\}$ is a survival probability of the object.

The expected utility of the insurance company

$$V(x, a) = \sum_y v(y)p(y), \qquad (19)$$

where $p(y)$ is a probability of profit y,
$v(y)$ is an utility function of the profit y.
Suppose that
$y = d(x)$
and

$$d(x) = \begin{cases} ax, & \text{if } \delta = 1 \\ -(1-a)x & \text{if } \delta = 0 \end{cases}. \qquad (20)$$

Here insurance policy x is defined by the owner of object which maximizes his utility (15) depending on the rate of insurance charge a. The equilibrium between interests of the company and the customer is achieved when both insurance policy x and insurance charge a satisfies Nash conditions.

5.1.1 Search for Equilibrium

First we fix the initial values, the "Contract-Vector" (x^0, a^0). The transformed values, the "Fraud-Vector" (x^1, a^1), are obtained by maximizing the utilities $U(x,a)$ and $V(x,a)$ respectively. The maximization is performed under the assumption that a partner honors the contract $(x^0, a^0$

$$x^1 = arg\max_x U(x, a^0), \tag{21}$$

$$a^1 = arg\max_a V(x^0, a_)), \tag{22}$$

Formally, condition (22) transforms the vector
$w^n = (x^n, a^n) \in B,\ n = 0,1,2,...$ into the vector w^{n+1}. To make expressions shorter denote this transformation by T

$$w^{n+1} = T(w^n),\ n = 0,1,2,... \tag{23}$$

One may obtain the equilibrium at the fixed point w^n, where

$$w^n = T(w^n). \tag{24}$$

The fixed point w^n exists, if the feasible set B is convex and all the profit functions are convex [2]. We obtain the equilibrium directly by iterations (23), if the transformation T is contracting [4]. If not, then we minimize the square deviation

$$\min_{w \in B} \parallel w - T(w) \parallel^2 . \tag{25}$$

The equilibrium is achieved, if the minimum (25) is zero. If the minimum (25) is positive then the equilibrium does not exist. That is a theoretical conclusion. In statistical modeling, some deviations are inevitable. Therefore, we assume that the equilibrium exists, if the minimum is not greater then modeling errors.

5.2 Single Company Insuring Multiple Objects

In this section the insurance model of multiple objects $i = 1, ..., n$ is reduced to that of a single object. Some companies apply "zero-one" insurance policies: $x_i = 0$ or $x_i = z_i$. In this case no equilibrium exist because only two discrete strategies can be applied[3]. If a number of customers is large, for example a set of car owners, one approximates discrete set by the continuous

[3]Theoretically one can transform the case into the convex one by introducing randomization, however that is not acceptable in the practical insurance.

one assuming that x is a sum of insurance policies of n customers that decided to insure their property $x = \sum_{i=1}^{n} x_i$. Assume as a first approximation that

$$x_i = z_i = 1. \tag{26}$$

Denote by $u(y)$ some cumulative utility function for a set of customers where variable y means their total wealth. In theory the cumulative utility $u(y)$ is determined by individual utilities $u_i(y_i)$. However, that is not a trivial computational problem. Statistical analysis of corresponding data is needed, too.

Under these assumptions the expected cumulative utility

$$U(x, a) = \sum_{y} u(y)p(y), \tag{27}$$

where $p(y)$ is a probability of the wealth y.

Suppose that total wealth
$y = c(x)$.
From (26)

$$c(x) = -al + (1 - a)(n - l), \tag{28}$$

where $x = n$ is a number customers that decided to insure their property and l is a number of survives.

From the assumption that survival probabilities p of all the objects[4] are equal and independent follows the binomial distribution. One approximates the binomial distribution by the Poisson distribution if $\lambda = qn$, where $q = 1 - p$ is not very large and not very small. Then the probability that l of n objects survive and the rest $n - l$ do not

$$p(n, l, q) = e^{-\lambda} \frac{\lambda^l}{l!} \tag{29}$$

From here and assumption (26) the probability $p(y)$ of returned wealth $y = -la + (1 - a)(n - l)$ is

$$p(y) = p(n, l, q) = Py = -la + (1 - a)(n - l). \tag{30}$$

[4]It is assumed that each customer owns a single object of market value $z = 1$ and that there are just two feasible insurance policies $x = 1$ or $x = 0$.

This completes the definition of the expected cumulative utility $U(x, a)$.

The expected utility $V(x, a)$ of insurance company is defined in a similar way.

$$V(x, a) = \sum_y v(y)p(y), \tag{31}$$

where $p(y)$ is a probability of profit y,
$v(y)$ is an utility function of the profit y.

Suppose that
$y = d(x)$.
From (26)

$$d(x) = an - (1 - a)(n - l), \tag{32}$$

where $x = n$ is a number customers that decided to insure their property and l is a number of survives.

From here and assumption (26) the probability $p(y)$ of profit $y = la - (1 - a)(n - l)$

$$p(y) = p(n, l, q) = Py = la - (1 - a)(n - l). \tag{33}$$

This defines the expected utility function $V(x, a)$ for an insurance company.

Here insurance policy x is defined indirectly by the number of customers n that insure their of objects maximizing their expected cumulative utility (15) that depends on the rate of insurance charge a. This is a correct assumption if the cumulative utility function $u(y)$ represents individual utilities $u_i(y_i)$ well enough. Therefore definition of $u(y)$ is important part of model that represents multiple customers as a single one.

The equilibrium between interests of the company and the customer is achieved when both insurance policy x and insurance charge a satisfies Nash conditions. One obtains the Nash equilibrium using the same expressions as in the previous section.

5.3 Two Competing Companies Insuring Multiple Objects

Following the model of previous section we will approximate discrete set of insurance policies x_i by the continuous one and shall use similar expressions of the expected cumulative utility function $U(x, a)$ of multiple customers.

The expected utilities $V_j(x, a_j)$ of two competing insurance companies $j = 1, 2$ are defined as in the previous section.

$$V_j(x, a) = \sum_{y_j} v_j(y_j) p(y_j), \tag{34}$$

where $p(y_j)$ is a probability of profit y_j,
$v_j(y_j)$ is a utility of the profit y_j.

Suppose that
$y_j = d(x_j)$.
From (26)

$$d(x_j) = a_j l_j - (1 - a_j)(n_j - l_j), \tag{35}$$

where $x_j = n_j$is a number customers that decided to insure their property at the company $j$and l_j shows how many of them survive.

From here and assumption (26) the probability $p(y_j)$ of profit $y_j = l_j a_j - (1 - a_j)(n_j - l_j)$

$$p(y_j) = p(n_j, l_j, q) = Py = l_j a_j - (1 - a_j)(n_j - l_j). \tag{36}$$

This defines the expected utility function $V_j(x_j, a_j)$ for the insurance company j.

Note that insurance policies $x_j, \ j = 1, 2$ are defined indirectly by the number of customers n_j that insure their objects at the company j. It is supposed that customers maximize expected cumulative utility that depends on the rates of insurance charges $a_j, \ j = 1, 2$.

The equilibrium between interests of companies and customers is achieved if insurance policies x_j and insurance charges a_j satisfy Nash conditions. Here search for the Nash equilibrium is performed minimizing differences between the fraud and contract vectors.

5.3.1 Search for Equilibrium

First we fix the initial values, the "Contract-Vector" $(x_j^0, a_j^0, \ j = 1, 2)$. The transformed values, the "Fraud-Vector" $(x_j^1, a_j^1, \ j = 1, 2)$, are obtained by maximizing the utilities $U(x_j, a_j)$ and $V_j(x_j, a_j)$ respectively. The maximization is performed under the assumption that partners honor the contract

$(x_j^0, a_j^0,\ j = 1.2)$

$$(x_1^1, x_2^1) = arg \max_{x_1,x_2} U(x_1, x_2, a_1^0, a_2^0), \tag{37}$$

$$a_j^1 = arg \max_{a_j} V_j(x_j^0, a_j, a_l^0,\ l \neq j),\ j = 1, 2 \tag{38}$$

Formally, condition (22) transforms the vector
$w^n = (x_j^n, a_j^n,\ j = 1, 2) \in B,\ n = 0, 1, 2, ...$ into the vector w^{n+1}. To make expressions shorter denote this transformation by T

$$w^{n+1} = T(w^n),\ n = 0, 1, 2, ... \tag{39}$$

One may obtain the equilibrium at the fixed point w^n, where

$$w^n = T(w^n). \tag{40}$$

The fixed point w^n exists, if the feasible set B is convex and all the profit functions are convex [2]. We obtain the equilibrium directly by iterations (23), if the transformation T is contracting [4]. If not, then we minimize the square deviation

$$\min_{w \in B} \parallel w - T(w) \parallel^2 . \tag{41}$$

The equilibrium is achieved, if the minimum (25) is zero. If the minimum (25) is positive then the equilibrium does not exist. That is a theoretical conclusion. In statistical modeling, some deviations are inevitable. Therefore, we assume that the equilibrium exists, if the minimum is not greater then modeling errors.

The problem can be directly extended to M competing companies and N individually insured objects. The only obstacle is dimensionality of related optimization problem. For example, in this case one searches for equilibrium of four variables (x_1, x_2, a_1, a_2) where variables (x_1, x_2) are controlled by customers responding to insurance charges (a_1, a_2) that are defined by coresponding insurance companies.

6 Utility Functions

Utility functions $u(y)$ are different for different persons and organizations. An individual utility function is defined by a lottery

$L(A, B, p) = \{pA + (1-p)B\}$. Here p is the probability to win the best event A.
$(1-p)$ is the probability to get the worst one B. Denote by C the "ticket price" of this lottery. There are two possible decisions:

- keep the ticket money C,
- bay a ticket and risk losing this money while hoping to win a greater wealth A with probability p.

Denote by $p(C)$ a "hesitation" probability, when one cannot decide which decision to prefer. One defines the "hesitation" probability $p(C)$ by this condition

$$L(A, B, C, p(C)) = [C \approx \{p(C)A + (1 - p(C))B\}]. \tag{42}$$

Here the symbol $\approx$ denotes the "hesitation." If utilities $u(A) = 1$ and $u(B) = 0$, the utility of the "ticket" C is equal to the hesitation probability $u(C) = p(C)$ [1].

Suppose, for example, that event C is to keep all the investment capital, $y = 1$, in a safe; no risk, no profit. Assume that the event A means doubling the capital, $y = 2$. The event B means losing all the capital, $y = 0$.

Denote by $p(1)$ the hesitation probability. Then
$u(1) = u(0) + p(1)(u(2) - u(0))$. If $u(0) = 0$ and $u(2) = 1$ then the utility of the capital $u(1) = p(1)$. Here one obtained capital utilities at three points: $y = 0,\ y = 1$, and $y = 2$.

To define a reasonable approximation of the utility function $u(y)$, we need at least two additional points. For example, points $y = 0.5$ and $y = 1.5$. One defines the corresponding utilities by the hesitation probabilities $p(0.5)$ and $p(1.5)$. These are obtained by two hesitation lotteries

$$\begin{aligned} L(1.0, 0.0, 0.5, p(0.5)) = \\ [(y = 0.5) \approx \{p(0.5)(y = 1) + (1 - p(0.5))(y = 0)\}] \end{aligned} \tag{43}$$

and

$$\begin{aligned} L(2.0, 1.0, 1.5, p(1.5)) = \\ [(y = 1.0) \approx \{p(1.5)(y = 2.0) + (1 - p(1.5))(y = 1)\}]. \end{aligned} \tag{44}$$

Here one obtains utility values
$u(0) = 0,\ u(0.5) = p(0.5),\ u(1) = p(1),\ u(1.5) = u(1) + p(1.5)(u(2) -$

$u(1)$) $u(2) = 1$. The remaining capital utility values are defined by the linear interpolation

$$u(y) = u(y_i) + p(y_i)(u(y_{i+1}) - u(y_i)), \quad y_i \le y < y_{i+1}, \quad i = 0, 1, ..., 4. \tag{45}$$

In consulting offices, the "psychological tests" defining capital utilities are not always convenient. Then one of the four "typical" utility functions can be selected. The typical utility functions represent the risky, the average, the rich and the prudent persons. The selection depends on observable personal traits.

The same capital utility function (14) could be used for all customers. Then one defines customer differences by different border points, namely: $r_{prudent} < r_{average} < r_{rich} < r_{risky}$ (see Figure **??**).

7 Software Example

7.1 Running the Program

The software is on the web-site and is run by Internet

1. open the line 'Portfolio-GMJ2-Flexible' in the 'Global Optimization',
2. start the applet,
3. select investment parameters (see upper Figure 1),
4. define utility function (see bottom Figure 1),
5. click the 'count' label at the top of page,
6. read results of the optimal investment, (see upper Figure 2),
7. open the 'convergence' window (see botom Figure 2) which show how the best utility depends on the iteration number,

7.2 A Set of Utility Functions

Nine approximation points are used: $yk[i] = 0.25 * i, \; i = 0, ..., 8$. Four utility functions are available. They differ at the approximation points. They represent different persons.

prudent person : $f[0] = 0.0, \; f[1] = 0.3, \; f[2] = 0.5, \; f[3] = 0.7, \; f[4] = 0.8, \; f[5] = 0.85, \; f[6] = 0.9, \; f[7] = 0.95, \; f[8] = 1.0,$

risky person : $f[0] = 0.0, \; f[1] = 0.1, \; f[2] = 0.2, \; f[3] = 0.25, \; f[4] = 0.3, \; f[5] = 0.4, \; f[6] = 0.7, \; f[7] = 0.9, \; f[8] = 1.0,$

rich person : $f[0] = 0.0, \; f[1] = 0.125, \; f[2] = 0.25, \; f[3] = 0.325, \; f[4] = 0.5, \; f[5] = 0.625, \; f[6] = 0.75, \; f[7] = 0.875, \; f[8] = 1.0,$

average person $f[0] = 0.0, \; f[1] = 0.1, \; f[2] = 0.2, \; f[3] = 0.25, \; f[4] = 0.5, \; f[5] = 0.7, \; f[6] = 0.85, \; f[7] = 0.95, \; f[8] = 1.0.$

Users select one of them.

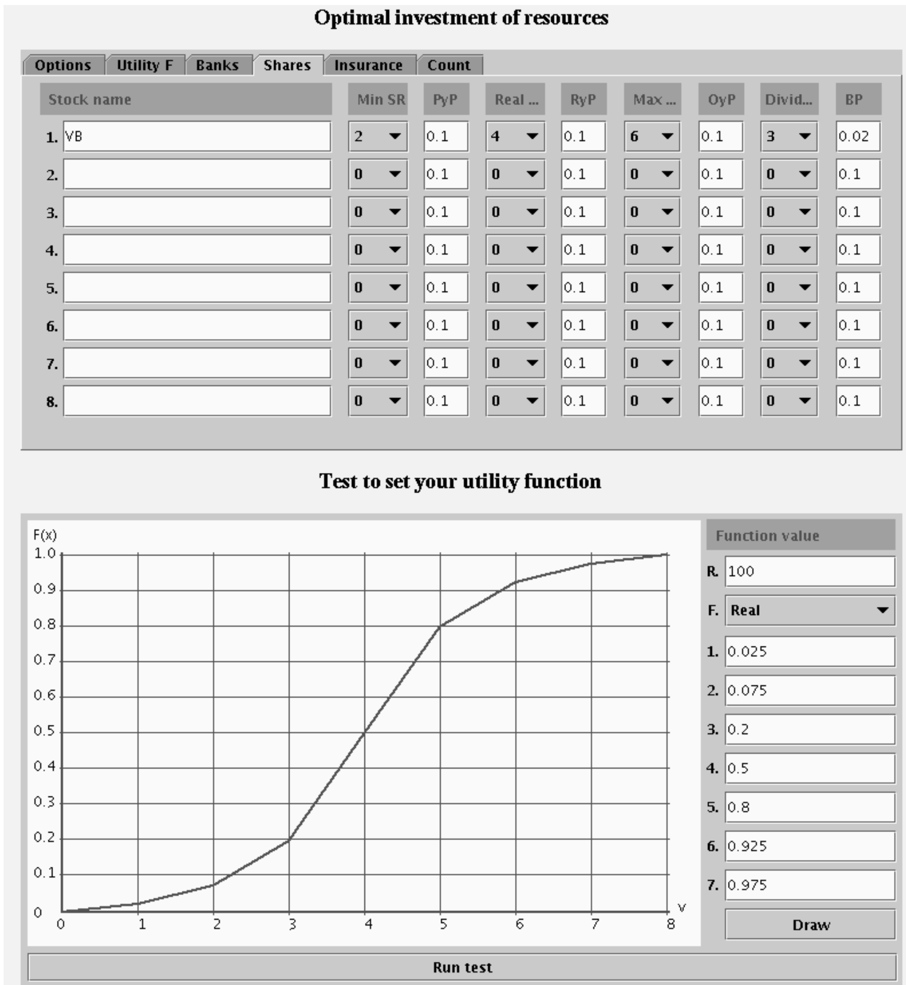

Figure 1: Optimal Investment, control windows.

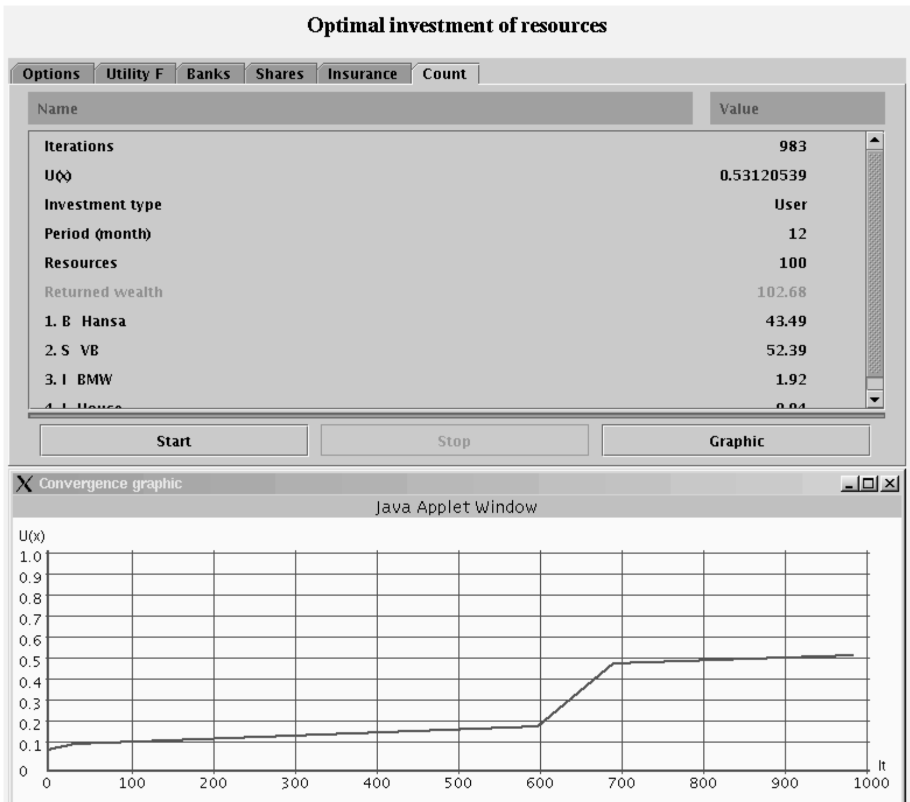

Figure 2: Optimal Investment, output windows.

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